Suffolk County Community College
Michael J. Grant Campus

Department of Mathematics

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MAT 106
Mathematics for Health Science

Final Exam: Solutions and Answers

Instructor:

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Problem 1. Vancomycin is an antibiotic recommended intravenously as a treatment of neonatal sepsis with the following dosages:

Gestational Age	Postnatal Age	Dose	Frequency
< 30 weeks	0 – 7 days	10 mg/kg	12 hourly
< 30 weeks	> 7 days	10 mg/kg	8 hourly
30 – 37 weeks	0 – 7 days	15 mg/kg	12 hourly
30 – 37 weeks	> 7 days	15 mg/kg	8 hourly
37 – 44 weeks	All ages	25 mg/kg	12 hourly

(Gestational age is computed as 40 weeks minus the number of weeks before due date at birth. For example, if a baby is born 2 weeks prematurely, its gestational age is 38 weeks.)

(1). The drug is available in the form of a 500 mg vial with powder that needs to be diluted into solution.

The solution is prepared according to the following protocol.

1. add 10 mL of sterile water for injections into the 500 mg vial;
2. withdraw 1 mL of the resulting solution;
3. further dilute this 1 mL to 10 mL with 0.9% Sodium Cloride;
4. discard any remaining solution from the vial immediately.

What is the concentration of the resulting solution?

Space for your solution:

After the whole vial's content is diluted in 10 mL, these 10 mL contain 500 mg of the pure drug. Consequently, the 1 mL of that solution that is used for the next step contains 50 mg of the pure drug. As these 50 mg are diluted further to 10 mL, the final concentration becomes $50/10 = 5$ mg/mL.

(2). What is the gestational age of a baby born 1 month prematurely?

Space for your solution:

If the baby is born 1 month prematurely, it has gestatinal age $40 - 4 = 36$ weeks.

(3). The baby considered in the previous problem developed sepsis by the 5 day since birth. It weighs 2 kg. What is the single dose of pure Vancomycin to be administered?

Space for your solution:

According to the general dosage requirements, this baby (gestatinal age 36 weeks and post-natal age 5 days) must be given 15 mg/kg every 12 hours. Given that the baby weighs 2 kg, the single dose is $15 \cdot 2 = 30$ mg of the drug.

(4). What is the single dose of the Vancomycin solution to be administered?

Space for your solution:

Since the solution contains 5 mg/mL, and the pure drug dose is 30 mg, we need to administer 6 mL of the solution.

Problem 2. Sickle cell anaemia is an autosomal recessive disorder. It affects erythrocytes (the red blood cells that transport oxygen). Individuals with two normal alleles have normal erythrocytes, but are easily infected with the malaria.

Those who have two defective alleles suffer from the anaemia. Their erythrocytes develop abnormally and may collapse when deoxygenated. However, malaria parasite cannot grow in those abnormal erythrocytes. Therefore people with anaemia are protected from malaria, but suffer from the effects of the erythrocyte defect.

Those who are heterozygous (i.e. are carriers: have one normal and one defective allele) have some sickling of erythrocytes, but do not suffer any ill effects from it, except when severely dehydrated or deprived of oxygen. In addition, malaria parasite cannot reproduce well within these their partially defective erythrocytes. Thus, heterozygous individuals tend to reproduce at a higher rate than those who have one of the two homozygous genotypes.

Compute all probabilities with at least four digits after the decimal.

(1). In a particular family, one parent is healthy and another one has sickle cell anaemia.

They had a child who also suffers from the anaemia. What is the probability that their next child will have sickle cell anaemia?

Space for your solution:

Based on the fact that these two parents have a child who suffers from sickle cell anemia, we can conclude that the healthy parent is a carrier of one sickle cell anemia allele. Therefore

$$P(\text{next child has sickle cell anemia}) = P\begin{pmatrix}\text{carrier parent}\\ \text{gives the}\\ \text{defective allele}\\ \text{to the next child}\end{pmatrix} = \frac{1}{2}.$$

(2). Sickle cell anaemia is estimated to occur in 1 in 500 African Americans. What are the frequencies of the normal and defective sickle cell anaemia alleles in the African American population?

Space for your solution:

Since the sickle cell anaemia is the recessive trait,

$$P\begin{pmatrix}\text{defective}\\ \text{allele}\end{pmatrix} = \sqrt{P\begin{pmatrix}\text{sickle cell}\\ \text{anaemia}\end{pmatrix}} = \sqrt{\frac{1}{500}} \approx 0.0447.$$

By the formula of probability of the complement event,

$$P\begin{pmatrix}\text{normal}\\ \text{allele}\end{pmatrix} = 1 - P\begin{pmatrix}\text{defective}\\ \text{allele}\end{pmatrix} \approx 1 - 0.0447 = 0.9553.$$

(3). Using the information from the previous subproblem, determine the probability of an African American to be a carrier of sickle cell anaemia.

Space for your solution:

Consider the Punnett square for the various sickle cell anaemia genotypes and their probabilities, for a child born to African American parents. Denote the normal sickle cell allele as S and the defective one as s.

	Father gave S, 0.9553	**Father gave s,** 0.0447
Mother gave S, 0.9553	SS, 0.9126	sS, 0.0427
Mother gave s, 0.0447	Ss, 0.0427	ss, 0.002

(The entries in the above table are rounded, and therefore approximate.) Therefore

$$P\begin{pmatrix}\text{child is}\\ \text{heterozygous}\end{pmatrix} =$$

$$P\left(\begin{pmatrix}\text{child has}\\ \text{genotype}\\ \text{sS}\end{pmatrix} \cup \begin{pmatrix}\text{child has}\\ \text{genotype}\\ \text{Ss}\end{pmatrix}\right) = P\begin{pmatrix}\text{child has}\\ \text{genotype}\\ \text{sS}\end{pmatrix} + P\begin{pmatrix}\text{child has}\\ \text{genotype}\\ \text{Ss}\end{pmatrix} =$$

$$\approx 0.0427 + 0.0427 = 0.0854.$$

(4). In a particular family, one parent is a healthy African American and another one has sickle cell anaemia. Determine the probability of their child having sickle cell anaemia.

Space for your solution:

Using the results of the previous subproblem and the formula of total probability,

$$P(\text{child has sickle cell anaemia}) =$$

$$P\begin{pmatrix}\text{healthy}\\ \text{parent}\\ \text{is not a}\\ \text{carrier}\end{pmatrix} \cdot P\left(\begin{matrix}\text{child has}\\ \text{sickle cell}\\ \text{anaemia}\end{matrix}\,\middle|\,\begin{matrix}\text{healthy}\\ \text{parent}\\ \text{is not a}\\ \text{carrier}\end{matrix}\right) + P\begin{pmatrix}\text{healthy}\\ \text{parent}\\ \text{is a}\\ \text{carrier}\end{pmatrix} \cdot P\left(\begin{matrix}\text{child has}\\ \text{sickle cell}\\ \text{anaemia}\end{matrix}\,\middle|\,\begin{matrix}\text{healthy}\\ \text{parent}\\ \text{is a}\\ \text{carrier}\end{matrix}\right)$$

$$= P\left(\begin{matrix}\text{African}\\ \text{American}\\ \text{is a carrier}\end{matrix}\,\middle|\,\begin{matrix}\text{African}\\ \text{American}\\ \text{is healthy}\end{matrix}\right) \cdot \frac{1}{2} = \frac{P\begin{pmatrix}\text{African}\\ \text{American}\\ \text{is a carrier}\end{pmatrix}}{2 \cdot P\begin{pmatrix}\text{African}\\ \text{American}\\ \text{is healthy}\end{pmatrix}} \approx \frac{0.0854}{2 \cdot (0.0854 + 0.9126)} \approx 0.0428.$$

Problem 3. Mammogram is an X-ray imaging of human breast, used for diagnosis or screening of breast cancer. Approximately 1 in 8 U.S. women (about 12.4%) will develop invasive breast cancer over the course of her lifetime: $P(\text{breat cancer}) = 12.4\%$. The following parameters can be defined for any diagnostic procedure. They are listed below with their estimated values for mammogram, based on *Saving Women's Lives: Strategies for Improving Breast Cancer Detection and Diagnosis.*

- sensitivity of the test (also called the "true positive" rate)

$$P\left(\begin{matrix}\text{positive}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{breast}\\\text{cancer}\end{matrix}\right) = 90\%$$

  (estimates range from 83 to 95 percent);

- specificity of the test (also called the "true negative" rate)

$$P\left(\begin{matrix}\text{negative}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{no breast}\\\text{cancer}\end{matrix}\right) = 95\%$$

  (estimates range from 90 to 98 percent);

- type I error probability (also called the "false positive" rate)

$$P\left(\begin{matrix}\text{positive}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{no breast}\\\text{cancer}\end{matrix}\right) = 1 - (\text{specificity}) = 5\%;$$

- type II error probability (also called the "false negative" rate)

$$P\left(\begin{matrix}\text{positive}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{no breast}\\\text{cancer}\end{matrix}\right) = 1 - (\text{sensitivity}) = 10\%;$$

Sensitivity and specificity of a mammogram depend on age of the patient and type of breast tissue. The above are the overall rates that should be adjusted in any specific case to take into account the particulars of the patient being tested.

(1). Find the probability that a random U. S. woman having a mammogram will get a positive mammogram result.

Space for your solution:

$$P\begin{pmatrix}\text{positive}\\\text{mammogram}\end{pmatrix} = \boxed{\text{formula of total probability}} =$$

$$P\begin{pmatrix}\text{breast}\\\text{cancer}\end{pmatrix} \cdot P\left(\begin{matrix}\text{positive}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{breast}\\\text{cancer}\end{matrix}\right) + P\begin{pmatrix}\text{no breast}\\\text{cancer}\end{pmatrix} \cdot P\left(\begin{matrix}\text{positive}\\\text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{no breast}\\\text{cancer}\end{matrix}\right)$$

$$= (\text{prevalence}) \cdot (\text{sensitivity}) + \big(1 - (\text{prevalence})\big)(\text{false positive rate})$$

$$= 12.4\% \cdot 90\% + (1 - 12.4\%) \cdot 5\% = 15.5\%.$$

(2). Positive predictive value of the mammogram test is defined as

$$P\left(\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\,\middle|\,\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\right).$$

Use the Bayes theorem and the previously mentioned data to find positive predictive value of a mammogram test for U.S. women.

Space for your solution:

$$P\left(\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\,\middle|\,\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\right) = \boxed{\text{Bayes formula}} =$$

$$\frac{P\left(\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\right)\cdot P\left(\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\right)}{P\left(\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\right)\cdot P\left(\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\right) + P\left(\begin{matrix}\text{no breast}\\ \text{cancer}\end{matrix}\right)\cdot P\left(\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\,\middle|\,\begin{matrix}\text{no breast}\\ \text{cancer}\end{matrix}\right)}$$

$$= \frac{(\text{prevalence})\cdot(\text{sensitivity})}{(\text{prevalence})\cdot(\text{sensitivity}) + \big(1-(\text{prevalence})\big)(\text{false positive rate})}$$

$$= \frac{12.4\%\cdot 90\%}{12.4\%\cdot 90\% + (1-12.4\%)\cdot 5\%} = 72\%.$$

This computation readily yields

$$P\left(\begin{matrix}\text{no breast}\\ \text{cancer}\end{matrix}\,\middle|\,\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\right) = 1 - P\left(\begin{matrix}\text{breast}\\ \text{cancer}\end{matrix}\,\middle|\,\begin{matrix}\text{positive}\\ \text{mammogram}\end{matrix}\right) = 28\%.$$

Confusing this latter probability (which may be rather high) with the probability computed in the previous sub-problem is sometimes called the “prosecutor’s fallacy”.