Suffolk County Community College
Michael J. Grant Campus
Department of Mathematics

Thursday, May 9, 2024 (expanded version)

MAT 106
Mathematics for Health Science

Final Exam: Solutions and Answers

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Problem 1. ST-Elevation Myocardial Infarction (STEMI) is a very serious type of heart attack during which one of the heart's major arteries is blocked. To prevent clots and improve blood flow to heart, 12 USP units/kg/hr (max 1000 units/hr per person) continuous IV infusion of heparin is indicated for STEMI patients.

(1). A STEMI patient weighs 70 Kg. Determine the dose of heparin in units per hour for this patient.

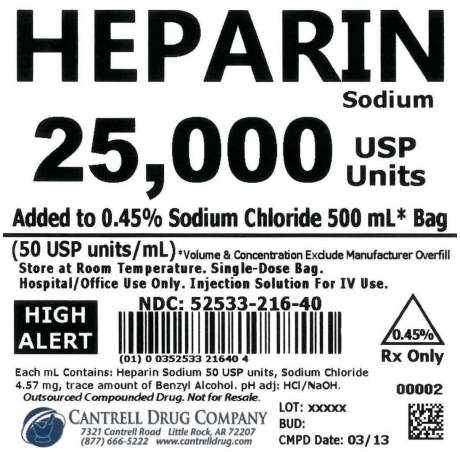
Space for your solution:

To determine the amount of heparin in USP units that needs to be administered each hour, we need to take into account the weight of the patient:

	USP Units	**kg**
Drug tissue concentration	12	1
Dose	x	70

Therefore, the patient needs $x = \frac{12 \cdot 70}{1} = 840$ USP Units of heparin per hour.

(2). Heparin is available in IV solution bag with label:



Determine the flow rate, in mL/hour, for the same patient.

Space for your solution:

	USP Units per hour	**mL per hour**
Concentration (see label)	50	1
Flow rate	840	x

Therefore $x = \frac{840 \cdot 1}{50}$ mL/h $= 16.8$ mL/h.

(3). If 15 gtt/mL tubing is used for administering the infusion, what should the flow rate be in drops per minute? (Round the answer to the nearest integer.)

Space for your solution:

$16.8 \text{ mL/h} = \frac{16.8 \cdot 15 \text{ drops}}{60 \text{ min}} = 4.2 \text{ drops/min} \approx 4 \text{ drops/min}$

(4). How long will this bag last?

Space for your solution:

Given that the bag contains 500 mL, and the infusion flow rate is 16.8 mL/h, the full bag will be used up in

$$\frac{500 \text{ mL}}{16.8 \text{ mL/h}} \approx 29.76 \text{ hr} \approx 29 \text{ hours } 46 \text{ min.}$$

Problem 2. This problem will introduce you to the *Rhesus Factor*.

Phenotype. An individual either has, or does not have, the *Rhesus factor D antigen* [1] on the surface of their erythrocytes. These two phenotypes are usually indicated by Rh+ (does have the the Rh(D) antigen) or Rh- (does not have the Rh(D) antigen) suffix appended to the ABO blood type.

Distribution. For example, the most typical blood type in the United States is O (Rh+) with 39% of the population having it; the least typical is AB (Rh-) shared only by 1% of Americans. In the African American population, 7% have the Rh- phenotype. Among the Americans of European descent, 16% have the Rh- phenotype.

Genetics. Rhesus factor is controlled by one gene [2], called Rh(D), located on chromosome one. The Rh- phenotype is recessive; the Rh+ phenotype is dominant.

Clinical Significance. An Rh- individual can get *immunized* against the Rh(D) antigen. Immunization can generally occur only through blood transfusion or — for women only — through placental exposure when giving birth. Immunized individuals produce anti-D antibodies. A woman who is

- Rh-
- immunized against the Rh(D) antigen, and
- pregnant with an Rh+ fetus (which may happen only if the father is Rh+),

will pass her anti-D antibodies to her fetus through placenta. Those antibodies will agglutinate the erythrocytes of the fetus, resulting in a severe anaemia or even death. This condition is called *Rh D Hemolytic* [3] *disease of the newborn* [4].

(1). Suppose a pregnant woman had no prior pregnancies or blood transfusions. Determine the probability of her current pregnancy being complicated by the Hemolytic disease of the newborn.

Space for your solution:

That probability is zero since the woman — even if Rh- — did not have the possibility of getting immunized against the Rh(D) antigen.

[1] or *Rh(D) antigen* for short
[2] this is somewhat of an oversimplification
[3] literally: destroying blood cells
[4] also called the *Rhesus disease*

(2). How many alleles of the Rh(D) gene does one cell of each of the following types have?

- erythrocyte (red blood cell in the blood);
- neuron;
- sperm cell;
- ova cell.

Space for your solution:

Knowing that the Rh(D) gene is located on chromosome one, we can conclude that it is a part of the nuclear DNA. Furthermore, since chromosome one is among the first 22 chromosomes, the Rh(D) gene belongs to the autosomal genome, which is not sex-specific. Thus we get:

- **erythrocyte — 0:** an erythrocyte does not have nucleus and thus has no nuclear DNA;
- **neuron —2:** a neuron is a somatic cell with diploid genome, having two alleles of the Rh(D) gene;
- **sperm cell — 1:** a sperm cell is a haploid gamete, having only a single allele of the Rh(D) gene;
- **ova cell —1:** an ova cell is also a haploid gamete, having only a single allele of the Rh(D) gene.

(3). Suppose a pregnant Rh- woman has two children: one Rh+, and another one is Rh- [5]. Her children, as well as her pregnancy, are from the same partner. What is the probability that the current pregnancy will be complicated by the Hemolytic disease of the newborn?

Space for your solution:

The woman is immunized against the Rh(D) antigen based on the fact that she gave birth to an Rh+ child. Thus the fetus in her current pregnancy will develop the Hemolytic disease of the newborn if and only if the fetus is Rh+.

Woman's partner must be Rh(D)-heterozygous since the two of them had one Rh+ and one Rh- child. Therefore the probability of the fetus being Rh+ (and thus developing the disease) is $\frac{1}{2}$.

[5] and has neither prior terminated pregnancies, nor deceased children

(4). Determine the frequency of the Rh- and Rh+ alleles in African American population, assuming that this population is in the state of Hardy-Weinberg equilibrium in regards to Rh(D) alleles. Round the answer to the nearest whole percent.

Space for your solution:

Denote the frequency of the Rh- allele as n, and Rh+ as p.

$$7\% = P\begin{pmatrix}\text{African American}\\ \text{has phenotype Rh-}\end{pmatrix} = \boxed{\text{Rh- phenotype is recessive}} = P\begin{pmatrix}\text{African American}\\ \text{has genotype Rh- Rh-}\end{pmatrix}$$

$$= \boxed{\text{law of inheritance}} = P\begin{pmatrix}\text{African American got}\\ \text{Rh- from each parent}\end{pmatrix} = \boxed{\text{product rule for independent events}} =$$

$$P\begin{pmatrix}\text{African American}\\ \text{got Rh- from father}\end{pmatrix}\cdot P\begin{pmatrix}\text{African American}\\ \text{got Rh- from mother}\end{pmatrix} = \left(P\begin{pmatrix}\text{African American}\\ \text{passed Rh- to child}\end{pmatrix}\right)^2 = n^2.$$

Therefore $n = \sqrt{7\%} \approx 26\%$ and $p \approx 100\% - 26\% = 74\%$.

(5). Assuming that African American population is in the state of Hardy-Weinberg equilibrium in regards to Rh(D) alleles, determine the probabilities that an African American person is 1) Rh+ and Rh(D) heterozygous and 2) Rh+ and Rh(D) homozygous. Round the answer to the nearest whole percent.

Space for your solution:

The Punnett square for the Rh(D) genotypes of a child born of African American parents:

	Father gave Rh+, 74%	**Father gave Rh-,** 26%
Mother gave Rh+, 74%	Rh+ Rh+, 55%	Rh- Rh+, 19%
Mother gave Rh-, 26%	Rh+ Rh-, 19%	Rh- Rh-, 7%

yields:

$$P\begin{pmatrix}\text{African American}\\ \text{is Rh+ and}\\ \text{Rh(D) homozygous}\end{pmatrix} \approx 55\%.$$

$$P\begin{pmatrix}\text{African American}\\ \text{is Rh+ and}\\ \text{Rh(D) heterozygous}\end{pmatrix} = P\begin{pmatrix}\text{African American has}\\ \text{genotype Rh- Rh+ or}\\ \text{genotype Rh- Rh+}\end{pmatrix} =$$

$$= \boxed{\text{inclusion-exclusion formula for union of mutually exclusive events}} =$$

$$P\begin{pmatrix}\text{African American}\\ \text{has genotype Rh- Rh+}\end{pmatrix} + P\begin{pmatrix}\text{African American}\\ \text{has genotype Rh+ Rh-}\end{pmatrix} \approx 19\% + 19\% = 38\%.$$

(6). Using the results from the sub-problem (5), determine the probabilities that an Rh+ African American person is 1) Rh(D) heterozygous and 2) Rh(D) homozygous. Round the answer to the nearest whole percent.

Space for your solution:

The task before us is to conditionalize the probabilities found in the previous sub-problem:

$$P\left(\begin{array}{c}\text{African}\\ \text{American is}\\ \text{Rh(D) homozygous}\end{array}\middle|\begin{array}{c}\text{African}\\ \text{American}\\ \text{is Rh+}\end{array}\right) =$$

$$= \frac{P\left(\begin{array}{c}\text{African American}\\ \text{is Rh+ and}\\ \text{Rh(D) homozygous}\end{array}\right)}{P\left(\begin{array}{c}\text{African}\\ \text{American}\\ \text{is Rh+}\end{array}\right)} \approx \frac{55\%}{55\% + 19\% + 19\%} \approx 59\%,$$

$$P\left(\begin{array}{c}\text{African}\\ \text{American is}\\ \text{Rh(D) heterozygous}\end{array}\middle|\begin{array}{c}\text{African}\\ \text{American}\\ \text{is Rh+}\end{array}\right) =$$

$$= \frac{P\left(\begin{array}{c}\text{African American}\\ \text{is Rh+ and}\\ \text{Rh(D) heterozygous}\end{array}\right)}{P\left(\begin{array}{c}\text{African}\\ \text{American}\\ \text{is Rh+}\end{array}\right)} \approx \frac{19\% + 19\%}{55\% + 19\% + 19\%} \approx 41\%.$$

(7). Suppose a European American woman has her second pregnancy from the father of her first child, and had no prior pregnancies [6] or blood transfusions. What is the probability of the current pregnancy being complicated by the Hemolytic disease of the newborn? Round the answer to the nearest whole percent. Assume that the father is Rh+ homozygous.

Space for your solution:

$$P\left(\begin{array}{c|c}\text{Hemolytic disease of the newborn} & \text{father is Rh+ homo-zygous}\end{array}\right) = \boxed{\text{the first child and the fetus are Rh+}} = P\left(\text{mother is Rh-}\right) = 16\%.$$

(8). Same as the previous sub-problem, but assume that the father is Rh(D) heterozygous.

Space for your solution:

$$P\left(\begin{array}{c|c}\text{Hemolytic disease of the newborn} & \text{father hetero-zygous}\end{array}\right) = P\left(\left(\text{mother is Rh-}\right) \cap \left(\text{first child is Rh+}\right) \cap \left(\text{fetus is Rh+}\right) \,\middle|\, \text{father hetero-zygous}\right) =$$

$$= \boxed{\text{product rule for intersection of events}} =$$

$$P\left(\begin{array}{c|c}\text{mother is Rh-} & \text{father is hetero-zygous}\end{array}\right) \cdot P\left(\left(\text{first child is Rh+}\right) \cap \left(\text{fetus is Rh+}\right) \,\middle|\, \left(\text{mother is Rh-}\right) \cap \left(\text{father is hetero-zygous}\right)\right) =$$

$$= \boxed{\text{Rh(D) of the first child and the fetus are independent}} = P\left(\begin{array}{c|c}\text{mother is Rh-} & \text{father is hetero-zygous}\end{array}\right) \cdot$$

$$P\left(\begin{array}{c|c}\text{first child is Rh+} & \left(\text{mother is Rh-}\right) \cap \left(\text{father is hetero-zygous}\right)\end{array}\right) \cdot P\left(\begin{array}{c|c}\text{fetus is Rh+} & \left(\text{mother is Rh-}\right) \cap \left(\text{father is hetero-zygous}\right)\end{array}\right)$$

$$= \boxed{\text{mother's and father's Rh(D) genotypes are independent}} = 16\% \cdot 50\% \cdot 50\% = 4\%.$$

[6] meaning pregnancies from other partners

(9). Suppose a European American woman has her second pregnancy from the African American father of her first child, and had no prior pregnancies [7] or blood transfusions. What is the probability of the current pregnancy being complicated by the Hemolytic disease of the newborn? Use results from sub-problems (5), as needed. Round the answer to the nearest whole percent.

Space for your solution:

The Hemolytic disiease may only happen if the father is Rh+ and the mother is Rh-. Thus:

$$P(\text{ Hemolytic disease of the newborn }) = \boxed{\text{formula of total probability}} =$$

$$P\begin{pmatrix}\text{father}\\ \text{is Rh+}\\ \text{hetero-}\\ \text{zygous}\end{pmatrix} \cdot P\left(\begin{array}{c|c}\text{Hemolytic} & \text{father}\\ \text{disease} & \text{is Rh+}\\ \text{of the} & \text{hetero-}\\ \text{newborn} & \text{zygous}\end{array}\right) + P\begin{pmatrix}\text{father}\\ \text{is Rh+}\\ \text{homo-}\\ \text{zygous}\end{pmatrix} \cdot P\left(\begin{array}{c|c}\text{Hemolytic} & \text{father}\\ \text{disease} & \text{is Rh+}\\ \text{of the} & \text{homo-}\\ \text{newborn} & \text{zygous}\end{array}\right)$$

$$= \boxed{\text{solution of sub-problem (5)}} =$$

$$\approx 38\% \cdot P\left(\begin{array}{c|c}\text{Hemolytic} & \text{father}\\ \text{disease} & \text{is Rh+}\\ \text{of the} & \text{hetero-}\\ \text{newborn} & \text{zygous}\end{array}\right) + 55\% \cdot P\left(\begin{array}{c|c}\text{Hemolytic} & \text{father}\\ \text{disease} & \text{is Rh+}\\ \text{of the} & \text{homo-}\\ \text{newborn} & \text{zygous}\end{array}\right)$$

$$= \boxed{\text{solutions of sub-problems (7) and (8)}} =$$

$$\approx 38\% \cdot 4\% + 55\% \cdot 16\% \approx 10\%.$$

[7] meaning pregnancies from other partners

(10). Using the Hardy-Weinberg principle, explain the difference in Rh(D) allele frequencies in African American and in European American populations.

Space for your solution:

The detrimental effect of Rh- allele on fertility of Rh- women must have been compensated by reproductive advantage, relative to Rh+ homozygous individuals, of Rh- men and Rh(D) heterozygous individuals. That advantage must have been higher in Europe than in Africa when the ancestors of the modern Americans evolved in those two regions.

The specific nature of this theoretically-certain-to-exist advantage is not known at the present time. However, there are studies [a] indicating that Rh(D) heterozygous individuals may be better protected against a decrease in psycho-motor performance associated with toxoplasma infection. If so, the geographical difference in Rh(D) allele frequencies may be reflective of a (hypotetical) higher rate of toxoplasmosis in the ancestors of European population. That, in turn, may have been a consequence of their migration from Africa through the Fertile Crescent. (The Fertile Crescent was the cradle of grain-based agriculture that made domestication of cats beneficial to humans [b]. Cats, on the other hand, are the definitive host of the *toxoplasmosa gondii* parasite and the key element in that parasite's life cycle. Domesticated cats spread from Egypt to Roman Empire around the turn of the first millennium, and from there on — to the rest of Europe.)

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[a] Flegr, Jaroslav: *Heterozygote Advantage Probably Maintains Rhesus Factor Blood Group Polymorphism*; PLoS One. 2016; 11(1): e0147955
`https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4728066/`

[b] Driscoll, Carlos A. et al.: *The Taming of the Cat*; Scientific American (2009): 71-72.
`https://www.researchgate.net/publication/253955800_The_Taming_of_the_Cat`

(11). Suppose a European American woman has her third pregnancy from the African American father of her first two children, and had no prior pregnancies or blood transfusions. Both of her children are healthy, i.e. did not suffer from the Hemolytic disease of the newborn. What is the probability of the current pregnancy being complicated by the Hemolytic disease of the newborn? Use results from all previous sub-problems, as needed. Round the answer to the nearest whole percent.

Space for your solution:

Denote : M_- = mother is Rh-, M_+ = mother is Rh+, F_-^- = father is Rh-,
F_-^+ = father is Rh(D) heterozygous, F_+^+ = father is Rh+ homozygous,
2ndOK = second child does not have the Hemolytic disease of the newborn.

The a-posteriori probability of the only combination of the parents' genotypes that may result in the Hemolytic disease of the newborn for the third child is:

$$P\left(\begin{matrix}M_-\\F_-^+\end{matrix}\,\middle|\,\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\right) = \boxed{\text{Bayes' formula}} =$$

$$= \frac{P\left(\begin{matrix}M_-\\F_-^+\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_-\\F_-^+\end{matrix}\right)}{\begin{matrix}P\left(\begin{matrix}M_+\\F_-^+\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_+\\F_+^+\end{matrix}\right) + P\left(\begin{matrix}M_+\\F_-^+\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_+\\F_-^+\end{matrix}\right) + P\left(\begin{matrix}M_+\\F_-^-\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_+\\F_-^-\end{matrix}\right) +\\ P\left(\begin{matrix}M_-\\F_+^+\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_-\\F_+^+\end{matrix}\right) + P\left(\begin{matrix}M_-\\F_-^+\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_-\\F_-^+\end{matrix}\right) + P\left(\begin{matrix}M_-\\F_-^-\end{matrix}\right)\cdot P\left(\begin{matrix}\text{2nd}\\\text{OK}\end{matrix}\,\middle|\,\begin{matrix}M_-\\F_-^-\end{matrix}\right)\end{matrix}}$$

$$= \boxed{\text{solutions of sub-problems (4), (5),}} =$$

$$\approx \frac{16\%\cdot 38\%\cdot 75\%}{\begin{matrix}84\%\cdot 55\%\cdot 100\% + 84\%\cdot 38\%\cdot 100\% + 84\%\cdot 7\%\cdot 100\% +\\ 16\%\cdot 55\%\cdot 0\% + 16\%\cdot 38\%\cdot 75\% + 16\%\cdot 38\%\cdot 100\%\end{matrix}} \approx 5\%.$$

Omitting the conditioning for brevity in the above probability, we get:

$$P\left(\begin{matrix}\text{third child}\\\text{has Hemolytic}\\\text{disease of the}\\\text{newborn}\end{matrix}\right) = P\left(M_-\cap F_-^+\cap\left(\begin{matrix}\text{first or}\\\text{second}\\\text{child}\\\text{is Rh+}\end{matrix}\right)\cap\left(\begin{matrix}\text{fetus}\\\text{is}\\\text{Rh+}\end{matrix}\right)\right) =$$

$$P\left(M_-\cap F_-^+\right)\cdot P\left(\begin{matrix}\text{first or}\\\text{second}\\\text{child}\\\text{is Rh+}\end{matrix}\,\middle|\,M_-\cap F_-^+\right)\cdot P\left(\begin{matrix}\text{fetus}\\\text{is}\\\text{Rh+}\end{matrix}\,\middle|\,M_-\cap F_-^+\right) \approx 5\%\cdot 75\%\cdot 50\% \approx 2\%.$$