Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 101
A Survey of Mathematical Reasoning

Final Exam: Solutions and Answers

Instructor:

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Existential presupposition is in effect for "some x are y" statements. The proposition "some cats are smart" means that the universe of discourse contains at least one cat who is smart. Existential presupposition is not in effect for "all x are y" statements. The proposition "all unicorns are mammals" means that every unicorn existing in the universe of discourse must be a mammal, but it does not assume that unicorns exists.

Problem 1. Consider the argument: *People aren't dogs. Some people understand logic. Therefore, no dog understands logic.*

(1). Identify the assumption and the conclusion of this argument. (Use any necessary logical connectors that may be implicit.)

Space for your solution:

Assumption (the conjunction $\wedge$ was implicit in the original):

$$\textit{(some people understand logic)} \wedge \textit{(no people are dogs)}.$$

Conclusion: *(no dogs understand logic).*

(2). What kind of an argument is this: propositional, syllogistic or predicate logic?

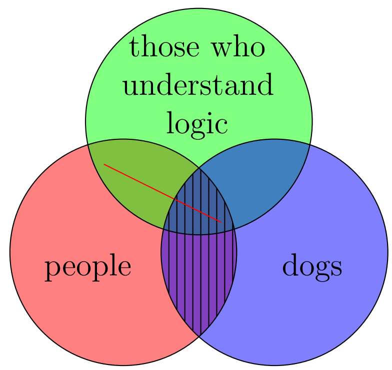
Space for your solution:

Since this argument involves no more than three categories, it is a syllogistic one. Since syllogistic arguments are a subset of predicate logic, this is also a predicate logic argument.

(3). Use the technique of analysis appropriate for this kind of an argument. (For a propositional argument: break it into atomic statements and logical connectors. For a syllogism: draw an Euler-Venn diagram and show the assumptions of this argument in the diagram. For a general first-order logic argument: determine objects, predicates and quantifiers, as well as atomic statements and logical connectors, that define the structure of this argument.)

Space for your solution:

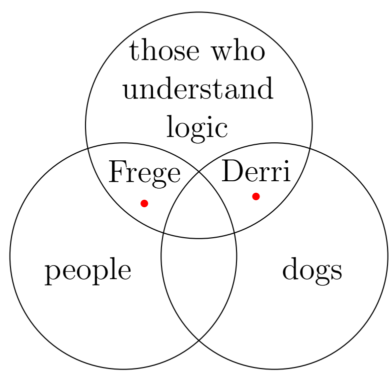
This is a syllogism and we can use an Euler-Venn diagram. In the diagram below, the red line represents the assumption "some people understand logic" and the vertical shading — "no people are dogs".



(4). Give a reason why this argument is valid or provide a counterexample showing that the argument is invalid.

Space for your solution:

Consider the universe of discourse that has only two objects "Frege" and "Derri". Frege is a person who understands logic, and Derri is a dog who understands logic. This universe satisfies the assumptions of the argument, but violates its conclusion. Thus this universe is a counterexample for this argument and the argument is invalid.



Problem 2. Consider the argument: *All unicorns are mammals. All mammals are animals. Therefore, some unicorns are animals.*

(1). Identify the assumption and the conclusion of this argument. (Use any necessary logical connectors that may be implicit.)

Space for your solution:

Assumption (the conjunction $\wedge$ was implicit in the original):

$$\textit{(all unicorns are mammals)} \wedge \textit{(all mammals are animals).}$$

Conclusion: *(some unicorns are animals).*

(2). What kind of an argument is this: propositional, syllogistic or predicate logic?

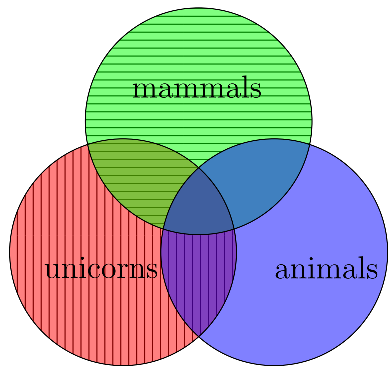
Space for your solution:

Since this argument involves no more than three categories, it is a syllogistic one. Since syllogistic arguments are a subset of predicate logic, this is also a predicate logic argument.

(3). Use the technique of analysis appropriate for this kind of an argument. (For a propositional argument: break it into atomic statements and logical connectors. For a syllogism: draw an Euler-Venn diagram and show the assumptions of this argument in the diagram. For a general first-order logic argument: determine objects, predicates and quantifiers, as well as atomic statements and logical connectors, that define the structure of this argument.)

Space for your solution:

This is a syllogism and we can use an Euler-Venn diagram to analyze it. In the diagram below, the vertical shading represents the assumption "all unicorns are mammals", and the horizontal — "all mammals are animals".



(4). Give a reason why this argument is valid or provide a counterexample showing that the argument is invalid.

Space for your solution:

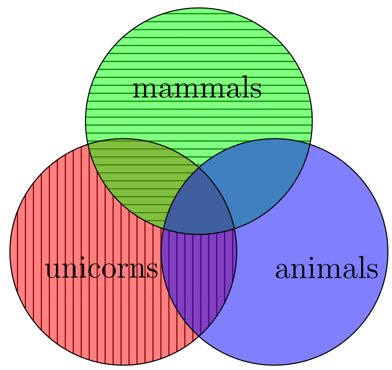
Consider the universe of discourse that has no objects at all. This universe satisfies the assumptions, but violates the conclusion. Thus the argument is invalid.

Problem 3. Consider the argument: *All unicorns are mammals. All mammals are animals. Therefore, all unicorns are animals.*

Give a reason why this argument is valid or provide a counterexample showing that the argument is invalid.

Space for your solution:

This is a syllogism and its Euler-Venn diagram is the same as the one for the previous problem. Let's use vertical shading to represent the assumption "all unicorns are mammals", and the horizontal — "all mammals are animals":



The above Euler-Venn diagram shows that any unicorn that may exist must exist within the category of animals. Thus the assumptions of the argument, if true, force the conclusion to be true as well. Therefore the argument is valid.

Problem 4. Consider the argument: *John or James was the murderer. James has an alibi for the time of the murder. Therefore John was the murderer.*

(1). Identify the assumption and the conclusion of this argument. (Use any necessary logical connectors that may be implicit.)

Space for your solution:

Assumption (the conjunction $\wedge$ was implicit in the original):

(John or James was the murderer) $\wedge$ *(James has an alibi for the time of the murder).*

Conclusion: *(John was the murderer).*

(2). What kind of an argument is this: propositional, syllogistic or predicate logic?

Space for your solution:

This argument can be understood by breaking it into atomic propositions connected by logical connectors. Thus it is a propositional argument. Since propositional logic a subset of predicate logic, this is also a predicate logic argument.

(3). Use the technique of analysis appropriate for this kind of an argument. (For a propositional argument: break it into atomic statements and logical connectors. For a syllogism: draw an Euler-Venn diagram and show the assumptions of this argument in the diagram. For a general first-order logic argument: determine objects, predicates and quantifiers, as well as atomic statements and logical connectors, that define the structure of this argument.)

Space for your solution:

This is a propositional argument and we can use its truth table to analyze its validity. Denote
A = "John was the murderer"
B = "James was the murderer".
Then the argument becomes:

$$\begin{array}{l} A \vee B \\ \neg B \\ \hline A \end{array}$$

Its truth table is:

A	B	$\big((A \vee B) \wedge \neg B\big) \Rightarrow A$
T	T	$\Big[\big((T \vee T) \wedge \neg T\big) \Rightarrow T\Big] = \Big[\big(T \wedge \neg T\big) \Rightarrow T\Big] = \Big[\big(T \wedge F\big) \Rightarrow T\Big] = [F \Rightarrow T] = T$
T	F	$\Big[\big((T \vee F) \wedge \neg F\big) \Rightarrow T\Big] = \Big[\big(T \wedge \neg F\big) \Rightarrow T\Big] = \Big[\big(T \wedge T\big) \Rightarrow T\Big] = [T \Rightarrow T] = T$
F	T	$\Big[\big((F \vee T) \wedge \neg T\big) \Rightarrow F\Big] = \Big[\big(T \wedge \neg T\big) \Rightarrow F\Big] = \Big[\big(T \wedge F\big) \Rightarrow F\Big] = [F \Rightarrow F] = T$
F	F	$\Big[\big((F \vee F) \wedge \neg F\big) \Rightarrow F\Big] = \Big[\big(F \wedge \neg F\big) \Rightarrow F\Big] = \Big[\big(F \wedge T\big) \Rightarrow F\Big] = [F \Rightarrow F] = T$

(4). Give a reason why this argument is valid or provide a counterexample showing that the argument is invalid.

Space for your solution:

The above truth table shows that the argument in question is a tautology, thus valid.

Problem 5. Consider the argument[1]: *The universe is a system of masses. Any finite system of masses has the barycenter. Any static system of masses with a barycenter is collapsed on its barycenter. If the universe were infinite and static, then every line of sight would end on some star, thus the night sky would be as bright as the sun. The universe is not collapsed. The night sky is not as bright as the sun. Therefore the universe is not static.*

(1). What kind of an argument is this: propositional, syllogistic or predicate logic?

Space for your solution:

This argument needs to be analyzed using logical connectors, objects and predicates. Thus it is a general predicate logic argument.

(2). Use the technique of analysis appropriate for this kind of an argument. (For a propositional argument: break it into atomic statements and logical connectors. For a syllogism: draw an Euler-Venn diagram and show the assumptions of this argument in the diagram. For a general first-order logic argument: determine objects, predicates and quantifiers, as well as atomic statements and logical connectors, that define the structure of this argument.)

Space for your solution:

Denote:

- the statement $L =$ (the night sky is as bright as the sun),
- the object $U =$ (The Universe),
- the predicate $m(x) = (x$ is a system of masses),
- the predicate $f(x) = (x$ is finite),
- the predicate $b(x) = (x$ has a barycenter),
- the predicate $c(x) = (x$ is collapsed),
- the predicate $s(x) = (x$ is static).

Using these notations, the original argument can be written as follows:

$$\begin{array}{l} m(U) \\ \forall x : m(x) \wedge f(x) \Rightarrow b(x) \\ \forall x : s(x) \wedge m(x) \wedge b(x) \Rightarrow c(x) \\ \neg f(U) \wedge s(U) \Rightarrow L \\ \neg c(U) \\ \neg L \\ \hline \neg s(U) \end{array}$$

[1]See Olber's paradox in *A Brief History of Time* by Stephen Hawking, page 58.

(3). Give a reason why this argument is valid or provide a counterexample showing that the argument is invalid.

Space for your solution:

This argument is valid and one can use, for instance, the following proof to demonstrate it:

1. $m(U)$ — assumption;
2. $\forall x : m(x) \wedge f(x) \Rightarrow b(x)$ — assumption;
3. $\forall x : s(x) \wedge m(x) \wedge b(x) \Rightarrow c(x)$ — assumption;
4. $\neg f(U) \wedge s(U) \Rightarrow L$ — assumption;
5. $\neg c(U)$ — assumption;
6. $\neg L$ — assumption;
7. $m(U) \wedge f(U) \Rightarrow b(U)$ — follows from 2 via universal instantiation $\dfrac{\forall x : p(x)}{p(C)}$;
8. $s(U) \wedge m(U) \wedge b(U) \Rightarrow c(U)$ — follows from 3 via universal instantiation;
9. $f(U) \Rightarrow b(U)$ — follows from 1 and 7 via $\dfrac{\begin{array}{l} A \\ A \wedge B \Rightarrow C \end{array}}{B \Rightarrow C}$;
10. $\neg\big(m(U) \wedge s(U) \wedge b(U)\big)$ — follows from 5 and 8 via reduction to absurdium $\dfrac{\begin{array}{l} A \Rightarrow B \\ \neg B \end{array}}{\neg A}$;
11. $\neg m(U) \vee \neg s(U) \vee \neg b(U)$ — follows from 10 via De Morgan's law;
12. $\neg\big(\neg f(U) \wedge s(U)\big)$ — follows from 6 and 4 via reduction to absurdium;
13. $f(U) \vee \neg s(U)$ — follows from 12 via De Morgan's law;
14. $\neg s(U) \vee \neg b(U)$ — follows from 1 and 11 via disjunctive syllogism $\dfrac{\begin{array}{l} A \vee B \\ \neg B \end{array}}{A}$;
15. $\neg f(U) \vee \neg s(U)$ — follows from 9 and 14 via $\dfrac{\begin{array}{l} A \Rightarrow B \\ \neg B \vee C \end{array}}{\neg A \vee C}$;
16. $\big(f(U) \wedge \neg f(U)\big) \vee \neg s(U)$ — follows from 9 and 14 via distributive law;
17. $\neg s(U)$ — follows from 16 via disjunctive syllogism.