Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 007
Algebra I

Final Exam: Solutions and Answers

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Problem 1. Solve the system of linear equations:

$$\begin{cases} 2x + y = 5 \\ x + y = 2 \end{cases}.$$

Space for your solution:

(This solution follows the substitution method.)

$\begin{cases} 2x + y = 5 \\ x + y = 2 \end{cases}$ $\Leftrightarrow$ (using second equation, express y in terms of x and constants; then substitute that expression instead of y in the first equation) $\begin{cases} y = 2 - x \\ 2x + (2 - x) = 5 \end{cases}$ $\Leftrightarrow$ (open parentheses and combine the like terms in the second equation) $\begin{cases} y = 2 - x \\ x + 2 = 5 \end{cases}$ $\Leftrightarrow$ (find the value of x from the second equation and then substitute that value instead of x in the first equation) $\begin{cases} y = 2 - 3 = -1 \\ x = 3 \end{cases}$.

Problem 2. Simplify the fraction:

$$\frac{\sqrt[3]{24\ x^2 y^5}}{3\sqrt[3]{x}\ y}.$$

Space for your solution:

$$\frac{\sqrt[3]{24\ x^2 y^5}}{3\sqrt[3]{x}\ y} = \frac{\sqrt[3]{2^3 \cdot 3\ x^2 y^5}}{3\sqrt[3]{x}\ y} = \frac{2^{\frac{3}{3}} \cdot 3^{\frac{1}{3}}\ x^{\frac{2}{3}} y^{\frac{5}{3}}}{3\ x^{\frac{1}{3}}\ y} =$$

$$2^{\frac{3}{3}} \cdot 3^{\frac{1}{3}-1}\ x^{\frac{2}{3}-\frac{1}{3}} y^{\frac{5}{3}-1} = 2 \cdot 3^{\frac{1}{3}-\frac{3}{3}}\ x^{\frac{1}{3}} y^{\frac{5}{3}-\frac{3}{3}} = 2 \cdot 3^{-\frac{2}{3}}\ x^{\frac{1}{3}} y^{\frac{2}{3}} = 2\sqrt[3]{\frac{xy^2}{9}}.$$

Problem 3. Consider two points $(-1, 5)$ and $(1, 1)$ on the (x, y)-plane.

(1). Find the slope of the line that joins these two points.

Space for your solution:

In general, the slope of the line joining (x_0, y_0) and (x_1, y_1) is

$$\frac{y_1 - y_0}{x_1 - x_0}.$$

In our case $(x_0, y_0) = (-1, 5)$ and $(x_1, y_1) = (1, 1)$, therefore the general formula turns into

$$\frac{(1) - (5)}{(1) - (-1)} = \frac{1 - 5}{1 + 1} = \frac{-4}{2} = -2.$$

(2). Find the slope-intercept equation of the line that joins these two points.

Space for your solution:

In general, the slope of the line joiningis In general, an equation of the line joining the points (x_0, y_0) and (x_1, y_1) can be stated as

$$\frac{y - y_0}{x - x_0} = m,$$

where m is the slope of the line in question.
In our case $(x_0, y_0) = (-1, 5)$ and $(x_1, y_1) = (1, 1)$, therefore the general equation turns into

$$\tfrac{y-(5)}{x-(-1)} = -2 \quad \Leftrightarrow \quad \tfrac{y-5}{x+1} = -2$$

To get the slope-intercept equation of the line, we multiply the previous equation by $x + 1$:

$$y - 5 = -2(x + 1) \quad \Leftrightarrow \quad y - 5 = -2x - 2 \quad \Leftrightarrow \quad y = -2x + 3.$$

(3). Find the y-intercept of the line that joins these two points.

Space for your solution:

From the slope-intercept equation $y = -2x + 3$ obtained in the previous sub-problem, we can see that the y-intercept is 3.

Problem 4. Solve the equation:

$$\frac{2}{x-3} + \frac{4}{x+3} = \frac{4x}{x^2-9}.$$

Space for your solution:

We have to find the Least Common Multiple of the three polynomials $x-3$, $x+3$ and x^2-9. Since $x^2-9=(x-3)(x+3)$, the polynomial x^2-9 is the LCM. After we multiply the fractions above by their respective adjoint factors, we will get them all have the same denominator:

$$\frac{2}{x-3} + \frac{4}{x+3} = \frac{4x}{x^2-9} \quad \Leftrightarrow \quad \frac{2(x+3)}{(x-3)(x+3)} + \frac{4(x-3)}{(x+3)(x-3)} = \frac{4x}{x^2-9} \quad \Leftrightarrow \quad \frac{2(x+3)+4(x-3)-4x}{(x-3)(x+3)} = 0 \quad \Leftrightarrow$$

$$\frac{2x+6+4x-12-4x}{(x-3)(x+3)} = 0 \quad \Leftrightarrow \quad \frac{2x-6}{(x-3)(x+3)} = 0 \quad \Leftrightarrow \quad \begin{cases} 2x-6=0 \\ (x-3)(x+3) \neq 0 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} 2x=6 \\ (x-3)(x+3) \neq 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x=3 \\ (x-3)(x+3) \neq 0 \end{cases} \quad \Leftrightarrow \quad x \in \emptyset.$$

The last conclusion (no solution) is made because the conditions $x=3$ and $(x-3)(x+3) \neq 0$ are contradictory. Indeed, if we substitute $x=3$ into $(x-3)(x+3)$ we do get zero.